

Cosmic age test in inhomogeneous cosmological models mimicking Λ CDM on the light cone

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The possibility of reconstructing a spherically symmetric inhomogeneous Lemaître-Tolman-Bondi (LTB) model with Λ CDM observations has drawn much attention. Recently, an inhomogeneous LTB model having the same luminosity-distance and light-cone mass density of the homogeneous Λ CDM model was reconstructed. From the Wilkinson microwave anisotropy probe 7-year measurements together with other cosmological observations, we calculate the cosmic age at our position in this LTB model, and obtain a constraint $t_{LTB} < 11.7\text{Gyr}$ at 1σ confidence level. We find that this result is, although 2Gyr younger compared with the age of the homogeneous Λ CDM model, still within 1σ agreement with the constraint of cosmic age given by current astronomical measurements. We expect that in the future with the help of more advanced observations we can distinguish the reconstructed inhomogeneous LTB model from the homogeneous Λ CDM model.

I. INTRODUCTION

The problem of dark energy has become one of the most important issues of the modern cosmology since the observations of type Ia supernovae (SNe Ia) [1] first indicated that the universe is undergoing an accelerated expansion at the present stage (if assuming that the universe is described by the Friedmann-Lemaître-Robertson-Walker (FLRW) model). Many cosmologists believe that the identity of dark energy is the cosmological constant which fits the observational data very well. However, one still has reasons to dislike the cosmological constant since it suffers from the theoretical problems such as the “fine-tuning” and the “cosmic coincidence” puzzles [2]. Thus, a variety of proposals for dynamic dark energy have emerged. For example, the “scalar field” model [3] has been explored for a long time. Besides, the “holographic dark energy” models [4], which arise from the holographic principle of quantum gravity theory, has also attracted much attention.

There are also some other theoretical approaches to explain the current cosmic acceleration. For example, it is argued that the acceleration of the universe may signify the breakdown of Einstein’s theory of general relativity [5]. Another interesting idea [6] is based on the assumption that the universe is described by the spherically symmetric, inhomogeneous Lemaître-Tolman-Bondi (LTB) metric [7] [8] [9]. Recently, some authors further proposed a possibility of mimicking the cosmological constant in an inhomogeneous universe [10] [11] [12] [13]. The idea is that, since cosmological observations are limited on the light cone, it is possible to reconstruct an inhomogeneous cosmological model (indistinguishable from the homogeneous Λ CDM model) to explain the cosmic acceleration without a cosmological constant. The formalism of reconstruction was developed in [10] and was applied to the Λ CDM model in [11]. In [12], the authors constructed a spherically symmetric, inhomogeneous cosmological model reproducing the luminosity-distance and the light-cone mass density of Λ CDM model up to $z = 2$.

In this work, we focus on the reconstructed inhomogeneous cosmological model and investigate whether it is consistent with cosmological observations. Since in this model Λ CDM observations are exactly reconstructed on the light-cone, we should seek for some observational test *not limited on the light-cone* to distinguish it from the standard Λ CDM model.

Fortunately, we find that the age of the universe is an appropriate touchstone. The cosmic age, which depends on the evolution of the universe at a comoving position, contains information not limited on the light-cone (the age is

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uniform in the Λ CDM model but may be dependent on the position in an inhomogeneous cosmological model). Thus it may reveal the discrepancies between the Λ CDM model and the reconstructed inhomogeneous model. On the other hand, it is rather convenient to use the cosmic age to test the validity of a specific cosmological model. To do this one may just compare the result with the age of some old objects in our universe [14]. For example, to be consistent the age of our universe in our position must not be younger than the age of the oldest stellar in the Milky Way.

This paper is organized as follows. In Sec. II, following the procedure of [12], we introduce the inhomogeneous LTB model and explain how to reconstruct Λ CDM observations in this model. In Sec. III, we calculate the cosmic age at our position and compare the result with the age of some old objects. We summarize in Sec. IV.

II. LEMAÎTRE-TOLMAN-BONDI MODELS

In this section, following the procedure of [12], we explain how to reconstruct an inhomogeneous LTB model having the same luminosity distance and light-cone mass density of the homogeneous Λ CDM model.

The LTB models are spherically symmetric cosmological solutions to the Einstein equations with a dust stress-energy tensor. The general metric for the LTB models in a synchronous comoving coordinate takes the form,

$$ds^2 = -dt^2 + \frac{R'^2(r, t)}{1 + \beta(r)} dr^2 + R^2(r, t) d\Omega^2. \quad (1)$$

Following [10, 12], we use the prime superscript to denote $\partial/\partial r$, and the overdot to denote $\partial/\partial t$. Noticing that here r is a dimensionless coordinate, while $R(r, t)$ has the dimension of length. The Robertson-Walker metric can be recovered after performing $R(r, t) \rightarrow a(t)r$ and $\beta(r) \rightarrow -kr^2$. Solving the Einstein Equations one obtains the generalized ‘‘Friedmann Equations’’ for $R(r, t)$ and $\rho(r, t)$,

$$\dot{R}(r, t) = \sqrt{\beta(r) + \frac{\alpha(r)}{R(r, t)}}, \quad (2)$$

$$\kappa\rho(r, t) = \frac{\alpha'(r)}{R^2(r, t)R'(r, t)}. \quad (3)$$

And the photon radial null geodesic equation for $\hat{t}(r)$ is found directly from the LTB metric (we are only interested in the past light cone),

$$\frac{d\hat{t}(r)}{dr} = -\frac{R'(r, \hat{t}(r))}{\sqrt{1 + \beta(r)}}. \quad (4)$$

For convenience we denote quantities on the light cone by a hat. Thus we have,

$$R(r, \hat{t}(r)) \equiv \hat{R}; \quad R'(r, \hat{t}(r)) \equiv \hat{R}'; \quad \rho(r, \hat{t}(r)) \equiv \hat{\rho}. \quad (5)$$

Then let us focus on the reconstruction procedures. The method was discussed by Mustapha, Hellaby, and Ellis in 1997 [10], and was recently applied to the Λ CDM model in [11] [12] (A related formalism was developed by [15] [16]). Following their procedure, we take advantage of a coordinate freedom and rescale r so that *on the light cone*,

$$\hat{R}' = H_0^{-1} \sqrt{1 + \beta(r)}. \quad (6)$$

The corresponding coordinate transformation is $dr_1 = \frac{H_0 \partial \hat{R} / \partial r}{\sqrt{1 + \beta(r)}} dr$. In the following, for simplicity we will use r to denote r_1 . The redshift takes the form (see Sec. 2.3 in [10])

$$\frac{dz(r)}{dr} = (1 + z) \frac{\hat{R}'}{\sqrt{1 + \beta(r)}}. \quad (7)$$

Following the procedure of [12], we reconstruct $\hat{d}_L(z)$ and $\hat{\rho}(z)$ of the Λ CDM model. To do this we require that on the light cone,

$$(1 + z)^2 \hat{R}(z) = (1 + z) \int_0^z \frac{dz_1}{H_{\Lambda\text{CDM}}(z_1)}, \quad \hat{\rho}(z) dV_{LTB} = \rho_{M, \Lambda\text{CDM}} dV_{\Lambda\text{CDM}}, \quad (8)$$

where $\rho_{M,\Lambda\text{CDM}}(z)$ stands for mass density in the ΛCDM model, and $H_{\Lambda\text{CDM}}(z)$ stands for the Hubble constant in the ΛCDM model. These quantities take the forms,

$$\rho_{\Lambda\text{CDM}}(z) = 3\Omega_m M_p^2 H_{\Lambda\text{CDM}}(z)^2, \quad H_{\Lambda\text{CDM}} = H_0 \sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}. \quad (9)$$

For simplicity we only consider a flat ΛCDM model. We use the notations,

$$\Omega_m = \frac{\rho_m(0)}{\rho_C(0)}, \quad \Omega_\Lambda = \frac{\rho_\Lambda(0)}{\rho_C(0)}, \quad \rho_C = 3M_p^2 H_0^2, \quad \Omega_m + \Omega_\Lambda = 1. \quad (10)$$

Then from Eq. (8) it is straightforward to derive the following expressions,

$$\hat{R}(z) = \frac{1}{1+z} \int_0^z \frac{dz_1}{H_{\Lambda\text{CDM}}(z_1)}, \quad (11)$$

$$H_0^{-1} \hat{R}^2(z) \kappa \hat{\rho}(z) \frac{dr}{dz} = \frac{3\Omega_m H_0^2}{H_{\Lambda\text{CDM}}(z)} \left[\int_0^z \frac{dz_1}{H_{\Lambda\text{CDM}}(z_1)} \right]^2. \quad (12)$$

Furthermore, the following three equations can be obtained by solving the corresponding z , $\alpha(r)$ and $\beta(r)$ (Eqs. (19-21) in [12])

$$\frac{dz}{dr} = (1+z) \frac{H_{\Lambda\text{CDM}}(z)}{H_0}, \quad (13)$$

$$\frac{d\alpha}{dr} = \frac{1}{2} H_0^{-1} \hat{R}^2(z) \kappa \hat{\rho}(z) \left[\frac{1}{H_0 d\hat{R}/dr} \left(1 - \frac{\alpha}{\hat{R}} \right) + H_0 \frac{d\hat{R}}{dr} \right], \quad (14)$$

$$\beta(r) = \left(\frac{d\alpha}{dr} \frac{1}{H_0^{-1} \hat{R}^2 \kappa \hat{\rho}} \right)^2 - 1. \quad (15)$$

It should be emphasized that for these values of $\alpha(z)$ and $\beta(z)$, the corresponding LTB model *exactly reproduces* the luminosity-distance relation and light-cone mass density of the ΛCDM model. For convenience, in the following context we will use “LTB- ΛCDM model” to denote this reconstructed LTB model.

It should be stressed that since the LTB- ΛCDM model is reconstructed by mimicking ΛCDM model, it has the same luminosity-distance-redshift relation and light-cone mass-density-redshift relation as the homogeneous ΛCDM model. Thus when estimating some physical quantities of the LTB- ΛCDM model from cosmological measurements one can just use the obtained values of Ω_m and H_0 from the fit of the ΛCDM model (i.e. it is not necessary to impose a special constraint on the LTB- ΛCDM model). For example, in [12] the authors just take $\Omega_m=0.3$ to mimicking a flat ΛCDM with mass ratio $\rho_{m0}/\rho_{c0} = 0.3$.

III. COSMIC AGE AT OUR POSITION

In the first subsection, we derive the expression of the cosmic age in the LTB- ΛCDM model. Then in the second subsection, we estimate the age from cosmological observations, and compare it with that in the homogeneous ΛCDM model. Finally, in the third subsection, we discuss the validity of the LTB- ΛCDM model by considering the astronomical measurements of the age of some old objects in our universe.

A. Cosmic Age in the LTB- ΛCDM

First of all, we investigate the properties of Eqs.(2),(3),(11-15) at $r = 0$ (corresponds to our position in the universe). Since the function $R(r, t)$ represents the diameter distance, one expects $R(r, t) \rightarrow 0$ when r approaches zero. So $R(r, t)$ can be expanded near $r = 0$ as,

$$R(r, t) = R_1(t)r + \dots \quad (16)$$

Then from Eq. (14) we obtain

$$\alpha(r) = \alpha_3 r^3 + \dots, \quad \beta(r) = \beta_2 r^2 + \dots, \quad \dot{R}_1(t) = \sqrt{\beta_2 + \frac{\alpha_3}{R_1(t)}}. \quad (17)$$

From Eqs. (9)(12)(14), we expand $H_{\Lambda\text{CDM}}(z)$, $\hat{r}(z)$, $\hat{R}(z)$ to the leading order and substitute them into Eq. (14) to obtain α_3

$$H_{\Lambda\text{CDM}}(z) = H_0 + \dots, \quad r = z + \dots, \quad \hat{R}(z) = \frac{z}{H_0} + \dots, \quad \alpha_3 = \Omega_m H_0^{-1}. \quad (18)$$

The calculation of β_2 is also straightforward. From Eqs.(14)(15) it follows that

$$\beta(r) = \left(\frac{1}{2} \left[\frac{1}{H_0 d\hat{R}/dr} \left(1 - \frac{\alpha}{\hat{R}} \right) + H_0 \frac{d\hat{R}}{dr} \right] \right)^2 - 1. \quad (19)$$

Substituting $\hat{R}'(z) = H_0^{-1} + \hat{R}'_1 z + \hat{R}'_2 z^2 + \dots$ into Eq. (19), we obtain

$$\beta(z) = (H_0^2 \hat{R}'_1{}^2 - \Omega_m) z^2 + \dots = (H_0^2 \hat{R}'_1{}^2 - \Omega_m) r^2 + \dots \quad (20)$$

Notice that \hat{R}'_2 does not appear in the expression, so we just have to expand \hat{R}' to the first order. Using Eqs.(11)(13), it follows that,

$$\frac{d\hat{R}}{dr} = \frac{d\hat{R}}{dz} \frac{dz}{dr} = -\frac{1}{1+z} \frac{H_{\Lambda\text{CDM}}(z)}{H_0} \int_0^z \frac{dz_1}{H_{\Lambda\text{CDM}}(z_1)} + \frac{1}{H_0} = H_0^{-1} - H_0^{-1} z + \dots, \quad \Rightarrow \quad \hat{R}'_1 = H_0^{-1}. \quad (21)$$

Combining with Eq. (20) we obtain

$$\beta_2 = 1 - \Omega_m. \quad (22)$$

Next we calculate the age of the universe ($t_0 - t_{BB}$) at $r = 0$. From Eq. (17) it follows that,

$$t_0 - t_{BB} = \int_{t_{BB}}^{t_0} dt = \int_{R_1(t_{BB})}^{R_1(t_0)} \frac{dR_1(t)}{\dot{R}_1(t)}, \quad (23)$$

where the upper and lower bounds of the integral can be determined by Eq. (3),

$$R_1(t) = \left(\frac{3\alpha_3}{\kappa\rho(0,t)} \right)^{\frac{1}{3}}. \quad (24)$$

Finally, combining the above with Eqs.(9)(18), we obtain the age of the universe at $r = 0$ in the LTB- Λ CDM model,

$$t_0 - t_{BB} = \int_0^{H_0^{-1}} \frac{dR_1}{\sqrt{\Omega_\Lambda + \Omega_m H_0^{-1}/R_1}}. \quad (25)$$

B. Estimate t_{LTB} from Cosmological Observations

Next we estimate the cosmic age in the LTB- Λ CDM model and compare the result with that of the Λ CDM model. These two models are both determined by two parameters, the present matter ratio Ω_m , and the Hubble constant H_0 . For convenience, let us denote the cosmic age in the LTB- Λ CDM model and Λ CDM model at $r = 0$ as $t_{LTB}(\Omega_m, H_0)$ and $t_{\Lambda\text{CDM}}(\Omega_m, H_0)$. We have [we perform a parameter transformation $r = H_0^{-1}/(1+z)$ in $t_{\Lambda\text{CDM}}$]

$$t_{LTB}(\Omega_m, H_0) = \int_0^{H_0^{-1}} \frac{dR_1}{\sqrt{\Omega_m/(H_0 R_1) + 1 - \Omega_m}}, \quad (26)$$

$$t_{\Lambda\text{CDM}}(\Omega_m, H_0) = \int_0^{+\infty} \frac{dz}{H_\Lambda(z)(1+z)} = \int_0^{H_0^{-1}} \frac{dr}{\sqrt{\Omega_m/(H_0 r) + (1 - \Omega_m)(H_0 r)^2}}. \quad (27)$$

The only difference between $t_{\Lambda\text{CDM}}$ and t_{LTB} lies in the second term in the square root. Since $H_0 r < 1$ (the upper bound of the integral is H_0^{-1}), it is clearly that t_{LTB} is smaller than $t_{\Lambda\text{CDM}}$ given the same set of parameters.

Notice that $t_{LTB}(\Omega_m, H_0)$ is proportional to $1/H_0$. To prove this one can perform a coordinate transformation $\tilde{R}_1 = H_0 R_1$, which yields that,

$$t_{LTB}(\Omega_m, H_0) = \frac{1}{H_0} \int_0^1 \frac{d\tilde{R}_1}{\sqrt{\Omega_m/\tilde{R}_1 + 1 - \Omega_m}}. \quad (28)$$

So we have,

$$\frac{\partial}{\partial H_0} t_{LTB}(\Omega_m, H_0) = -\frac{1}{H_0} t_{LTB}(\Omega_m, H_0) < 0, \quad (29)$$

$$\frac{\partial}{\partial \Omega_m} t_{LTB}(\Omega_m, H_0) = -\frac{1}{2} \int_0^{H_0^{-1}} \left(\frac{\Omega_m}{H_0 r} + 1 - \Omega_m \right)^{-3/2} \left(\frac{1}{H_0 r} - 1 \right) dr < 0, \quad (30)$$

Clearly, $t_{LTB}(\Omega_m, H_0)$ has larger values with smaller values of Ω_m and H_0 parameters. One can easily verify that the conclusion is the same for $t_{\Lambda\text{CDM}}(\Omega_m, H_0)$.

Now we are ready to calculate the specific values of t_{LTB} with given values of H_0 and Ω_m . Here we refer to the result of the seven-year Wilkinson microwave anisotropy probe (WMAP) observations [17]. From the constraint from “WMAP7+BAO+ H_0 ” [17][18][19] the WMAP collaboration provides the best-fit values of Ω_Λ and H_0 together with their 1σ uncertainties,

$$\Omega_\Lambda = 0.728_{-0.016}^{+0.015}, \quad H_0 = 70.4_{-1.4}^{+1.3} \text{ km/s/Mpc}. \quad (31)$$

From their result we can put a constraint on t_{LTB} and $t_{\Lambda\text{CDM}}$ at 1σ confidence level (CL). The result is,

$$t_{LTB} = 11.4 \pm 0.3 \text{ Gyr}, \quad t_{\Lambda\text{CDM}} = 13.8 \pm 0.5 \text{ Gyr}. \quad (32)$$

It is found that t_{LTB} is about 2Gyr younger than $t_{\Lambda\text{CDM}}$. At the 1σ CL we obtain the upper limit of the age of the LTB- ΛCDM model $t_{LTB} < 11.7\text{Gyr}$, with the set of the smallest values of parameters $\Omega_m = 0.257$ and $H_0 = 69.0\text{km/s/Mpc}$ (In fact this result is overestimated since we ignore the degeneracy between Ω_m and H_0 . The upper limit value of t_{LTB} should be smaller, or at least as small as 11.7Gyr).

It should be stressed that to be strict the previous estimation of t_{LTB} is not appropriate, since we have assumed that the WMAP and baryon acoustic oscillations data could be used to constrain the LTB models. In fact, the issues of CMB and structure formation in the LTB scenario are rather complicated and have not been clearly investigated. To avoid this problem one can put constraints to Ω_m and H_0 from SNIa observations. The recent observations of SDSS-II (Sloan Digital Sky Survey II)[20] and Hubble Space Telescope[18] show that $\Omega_m > 0.224$, $H_0 > 70.6\text{km/s/Mpc}$ in 1σ CL. From their result we find a upper limit $t_{LTB} < 11.6\text{Gyr}$, which is the similar with our previous result $t_{LTB} < 11.7\text{Gyr}$.

C. Discussions of the Validity of the LTB- ΛCDM Model

The low limit to the cosmic age can be directly obtained from estimating the age of some old objects in our universe [21][22][23]. As an example, based on white dwarf cooling the authors of [22] get a result of $12.7 \pm 0.7\text{Gyr}$. Compared with the result of the previous subsection $t_{LTB} < 11.7\text{Gyr}$ it seems that the LTB- ΛCDM model is inconsistent with their measurements. However, one should not conclude hastily. The reason is that the result of [22] is subject to larger uncertainty. The uncertainty due to calculations of the white dwarf cooling is difficult to estimate, and in their result corresponding errors are not included. In fact, the authors of [22] argued that systematic uncertainties are likely to be at least as large as, if not larger than, the quoted statistical errors. So if indeed the uncertainty due to calculations of the white dwarf cooling is as large as the observational error, then the LTB- ΛCDM model is within 1σ agreement with observations.

The age based on evolution of compact binaries is somewhat lower. In [24][25] the authors give a result of $11.8 \pm 0.6\text{Gyr}$ and $11.10 \pm 0.67\text{Gyr}$, respectively. Moreover, the oldest known star in the Milky Way, HE 1523-0901, is reported to have an age of $13.2 \pm 2.7\text{Gyr}$ [26], for a lower limit of 10.5Gyr. Obviously, these measurements are all in consistent with t_{LTB} and $t_{\Lambda\text{CDM}}$ obtained with data from [17] at 1σ CL. Therefore, although the obtained t_{LTB} is about 2Gyr younger than $t_{\Lambda\text{CDM}}$, it is still in 1σ agreement with current astronomical observations, and we are not able to argue against the reconstructed LTB- ΛCDM model.

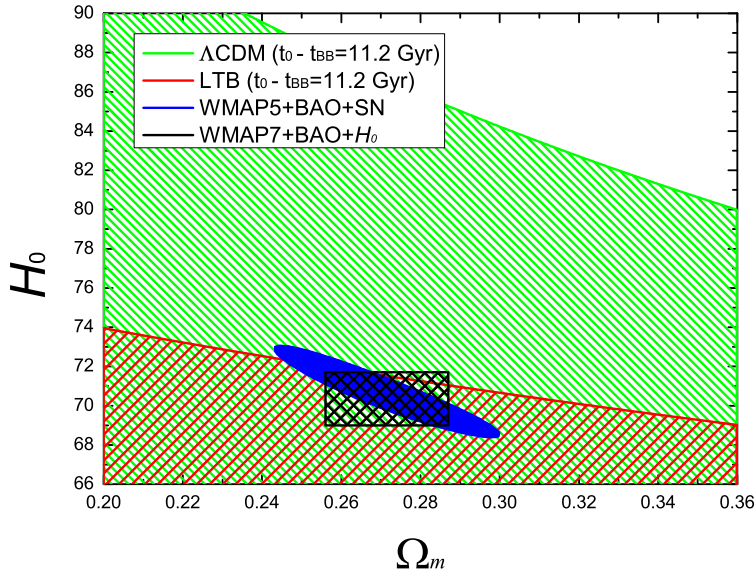


FIG. 1: Parameter space in Ω_m and H_0 plane. The green and red regions stand for $t_{\Lambda\text{CDM}} > 11.2\text{Gyr}$ and $t_{\text{LTB}} > 11.2\text{Gyr}$ respectively. The black shadow region represents the 1σ constraint to Ω_m and H_0 from the seven-year WMAP observations (degeneracy is ignored). The blue region is a 2σ constraint from a joint analysis from the Constitution supernovae sample, baryon acoustic oscillations and the five-year WMAP observations.

We show the situation in Fig. 1. The green and red regions represent parameters with cosmic age older than 11.2Gyr in ΛCDM and LTB- ΛCDM model, respectively. The black shadow region is a 1σ CL constraint from the seven-year WMAP observations [17] (we ignore degeneracy). The blue region is a 2σ CL constraint from a joint analysis performed in one of our previous works [27], in which we used the Constitution supernovae sample [28], the baryon acoustic oscillations [29] and the five-year WMAP observations [30]. Since current limit to the cosmic age from astronomical measurements generally gives a result $t < 11.2\text{Gyr}$, we plot the regions of $t_{\text{LTB}} < 11.2\text{Gyr}$ (red shadow) and $t_{\Lambda\text{CDM}} < 11.2\text{Gyr}$ (green shadow) in this figure. It is obvious that the ΛCDM perfectly passes the cosmic age test, while the overlap of the blue region, the black shadow region and the red shadow region implies that the LTB- ΛCDM model is also consistent with current observations.

Finally, at the end of this section, we stress that in this paper we only consider a particular LTB model - namely, the one with ΛCDM features. Other inhomogeneous models (such as the void models) may have larger age at the origin, and in these cases one would have to seek for other methods to test and identify them.

IV. SUMMARY

In this paper we calculate the cosmic age at $r = 0$ in the LTB- ΛCDM model, which reproduces the luminosity-distance and light-cone matter density of the homogeneous ΛCDM model. Using the constraints of Ω_m and H_0 from the seven-year WMAP observations combined with other cosmological observations, we get the upper limit $t_{\text{LTB}} < 11.7\text{Gyr}$ at 1σ CL. This result is about 2Gyr younger than the cosmic age in ΛCDM scenario. Since current astronomical measurements generally put a 1σ CL lower limit on the age of the universe of about 11.2Gyr, the LTB- ΛCDM model is still in 1σ agreement with all these observations. However, due to the relatively younger age the LTB- ΛCDM model might be disfavored by future observations.

Besides, even if the LTB- ΛCDM model successfully passes all the tests of future observations, there might be some other problems in this scenario. (The discussions of these complicated problems are beyond the scope of this paper.) The main reason is that it is difficult to fit this model into the larger framework of fundamental physics such as particle physics, general relativity, astrophysics, and cosmology. For example, in [12] the authors mentioned the theory of structure formation and the integrated Sachs-Wolfe effect: study of these issues is very difficult in the LTB models.

The topic of distinguishing the homogeneous ΛCDM model and the reconstructed inhomogeneous LTB- ΛCDM model is scientifically interesting and important, since it involves the question of the mysterious feature of dark

energy and whether the nearby region of the universe is homogeneous. This topic should be carefully investigated. In this paper we propose the possibility of distinguishing the LTB- Λ CDM scenario with the standard Λ CDM model by performing the cosmic age test. The procedure is convenient and straightforward. Although the result shows that the LTB- Λ CDM is still in 1σ agreement with current astronomical observations, since with the same set of parameters this model always has a younger age than the standard Λ CDM model it is possible to distinguish them from future observations. In all, the issue of using the cosmic age test to distinguish the reconstructed inhomogeneous LTB models from the homogeneous Λ CDM model is worth further investigation, and should be taken into consideration in future works, e.g., in the cases when people try to construct a new model in the LTB scenario to explain the apparent cosmic acceleration.

Acknowledgments

The authors would like to thank the anonymous referees for carefully examining our paper and providing us a number of important comments. This work was supported by the Natural Science Foundation of China under Grants No. 10535060/A050207, No. 10975172 and No. 10821504, and Ministry of Science and Technology 973 program under Grant No. 2007CB815401. SW also thanks the support from a graduate fund of the University of Science and Technology of China.

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- [1] A. G. Riess *et al.* *Astron. J.* **116**, 1009 (1998); S. Perlmutter *et al.* *Astrophys. J.* **517**, 565 (1999).
 - [2] S. Weinberg, *Rev. Mod. Phys.* **61**, 1 (1989).
 - [3] P. J. E. Peebles and B. Ratra, *Astrophys. J.* **325**, L17 (1988); C. Wetterich, *Nucl. Phys. B.* **302** 668 (1988); J. A. Frieman, C. T. Hill, A. Stebbins and I. Waga, *Phys. Rev. Lett.* **75**, 2077 (1995); M. S. Turner and M. J. White, *Phys. Rev. D* **56**, R4439 (1997); A. R. Liddle and R. J. Scherrer, *Phys. Rev. D.* **59**, 023509 (1998); I. Zlatev, L. M. Wang and P. J. Steinhardt, *Phys. Rev. Lett.* **82**, 896 (1999); P. J. Steinhardt, L. M. Wang and I. Zlatev, *Phys. Rev. D* **59**, 123504 (1999).
 - [4] M. Li, *Phys. Lett. B* **603**, 1 (2004), hep-th/0403127; Q. G. Huang and M. Li, *JCAP* **0408**, 013 (2004), astro-ph/0404229; Y. G. Gong, *Phys. Rev. D* **70**, 064029 (2004); Q. G. Huang and M. Li, *JCAP* **0503**, 001 (2005), hep-th/0410095; X. Zhang, *Int. J. Mod. Phys. D* **14**, 1597 (2005); M. Li, X. D. Li, S. Wang and X. Zhang, *JCAP* **0906**, 036 (2009); M. Li and Y. Wang, arXiv:1001.4466.
 - [5] S. M. Carroll, V. Duvvuri, M. Trodden and M. S. Turner, *Phys. Rev. D* **70**, 043528 (2004); S. Nojiri and S. D. Odintsov, *Phys. Rev. D* **68**, 123512 (2003); S. Capozziello, S. Carloni and A. Troisi, *Recent Res. Dev. Astron. Astrophys.* **1**, 625 (2003).
 - [6] H. Alnes, M. Amarzguoui and O. Gron, *Phys. Rev. D* **73**, 083519 (2006); Daniel J. H. Chung and A. E. Romano, *Phys. Rev. D* **74**, 103507 (2006); H. Alnes and M. Amarzguoui, *Phys. Rev. D* **74**, 103520 (2006); D. Garfinkle, *Class. Quant. Grav.* **23**, 4811 (2006); H. Alnes and M. Amarzguoui, *Phys. Rev. D* **75**, 023506 (2007); V. Marra, E. W. Kolb, S. Matarrese and A. Riotto, *Phys. Rev. D* **76**, 123004 (2007); K. Enqvist and T. Mattsson, *JCAP* **0702**, 019 (2007); M. Kasai, *Prog. Theor. Phys.* **117**, 1067 (2007); K. Enqvist, *Gen. Rel. Grav.* **40**, 451 (2008); C. Clarkson, B. Bassett and T. H.-C. Lu, *Phys. Rev. Lett.* **101**, 011301 (2008); R. R. Caldwell and A. Stebbins, *Phys. Rev. Lett.* **100**, 191302 (2008); J. Garcia-Bellido and T. Haugboelle, *JCAP* **0804**, 003 (2008); T. Biswas and A. Notari, *JCAP* **0806**, 021 (2008); J. Garcia-Bellido and T. Haugboelle, *JCAP* **0809**, 016 (2008); J. P. Zibin, A. Moss and D. Scott, *Phys. Rev. Lett.* **101**, 251303 (2008); S. Alexander, T. Biswas, A. Notari and D. Vaid, *JCAP* **0909**, 025 (2009); K. Bolejko and J. Stuart B. Wyithe, *JCAP* **0902**, 020 (2009); J. Garcia-Bellido and T. Haugboelle, *JCAP* **0909**, 028 (2009); S. February, J. Larena, M. Smith and C. Clarkson, arXiv:0909.1479.
 - [7] G. Lemaitre, *Ann. Soc. Sci. Bruxelles A* **53**, 51 (1933); English translation, with historical comments: *Gen. Rel. Grav.* **29**, 641 (1997)
 - [8] R. C. Tolman, *Proc. Nat. Acad. Sci. USA* **20**, 169 (1934); reprinted, with historical comments: *Gen. Rel. Grav.* **29**, 935 (1997)
 - [9] H. Bondi, *Mon. Not. Roy. Astron. Soc.* **107**, 410 (1947).
 - [10] N. Mustapha, C. Hellaby and G. F. R. Ellis, *Mon. Not. Roy. Astron. Soc.* **292**, 817 (1997).
 - [11] M.-N. C  l  rier, K. Bolejko and A. Krasinski, arXiv:0906.0905.
 - [12] E. W. Kolb and C. R. Lamb, arXiv:0911.3852.
 - [13] A. E. Romano, *JCAP* **1001** 004 (2010); A. E. Romano, arXiv:0912.2866.
 - [14] A.C.S. Friacas, J.S. Alcaniz, and J.A.S. Lima, *MNRAS*. **362**, 1295 (2005); S. Wang and Y. Zhang, *Phys. Lett. B* **669** 201(2008).
 - [15] D. J. H. Chung and A. E. Romano, *Phys. Rev. D* **74**, 103507 (2006).
 - [16] C. M. Yoo, T. Kai and K. I. Nakao, *Prog. Theor. Phys.* **120**, 937 (2008).
 - [17] E. Komatsu *et al.*, arXiv:1001.4538.
 - [18] A. G. Riess *et al.*, *Astrophys. J.* **699** 539 (2009).
 - [19] W. J. Percival *et al.*, arXiv:0907.1660.

- [20] R. Kessler *et al.*, *Astrophys. J. Suppl.* **185**, 32 (2009).
- [21] H. B. Richer *et al.*, *Astrophys. J.* **574** L151 (2002).
- [22] B. M. S. Hansen *et al.*, *Astrophys. J.* **574** L155 (2002).
- [23] L. M. Krauss and B. Chaboyer, *Science* **299** 65 (2003).
- [24] J. Kaluzny *et al.*, *Astron. Soc. Pac. Conf. Ser.* **265**, 155 (2002).
- [25] B. Chaboyer and L. M. Krauss, *Astrophys. J.* **567** L45 (2002).
- [26] A. Frebel *et al.*, *J. 2007, Astrophys. J. Lett.* **660** L117 (2007).
- [27] M. Li, X. D. Li and X. Zhang, arXiv:0912.3988.
- [28] M. Hicken *et al.*, *Astrophys. J.* **700**, 1097 (2009).
- [29] W. J. Percival *et al.*, *Mon. Not. Roy. Astron. Soc.* **381**, 1053 (2007).
- [30] E. Komatsu *et al.*, *Astrophys. J. Suppl.* **180**, 330 (2009).